PHY 113 (GENERAL PHYSICS 1 (MECHANICS))

1.0 UNITS AND DIMENSIONS

There are three important fundamental quantities in physics. These are length, mass, and time. The units of these quantities form the fundamental or basic units upon which most (though not all) other units depends. Other fundamental quantities are shown in the table below

S.I Fundamental Quantities and Units

QUANTITY	UNIT	UNIT ABBREVIATION
Length	Metre	m
mass	Kilogram	kg
time	Second	s
temperature	Kelvin	K
Electric current	Ampere	A
Amount of substance	Mole	mole
Luminous intensity	Candela	cd

1.2 Derived units

By simple combination of SI basic unit, we can obtain other useful units. They are called derived units. The unit of volume is obtained by multiplying three lengths m*m*m= m^3. Other important derived units are listed below.

QUANTITY	DERIVATION	UNITS
Area	Length * Breadth	m^2
Volume	Length * Breadth * Height	m^3
Density	Mass/ Volume	kg/m^3

Speed or Velocity	Distance/time	m/s
Acceleration	(Change in velocity)/time	m/s^2
Force	Mass * acceleration	kgm/s^2 (N)
Momentum	Mass * velocity = Force*Time	Ns
Pressure	Force/Area	N/m^2
Energy or work	Force * distance	Nm or joules

1.3 **Dimensions of Physical Quantities**

By the **dimension** of a physical quantity, we mean the way it is related to the fundamental quantities, mass, length and time. These are usually denoted by M, L and T respectively

- An area, Length* breadth, has dimension L*L or L^2
- Density, which is mass/volume, has dimension M/L^3 or ML^{-3}

The following are the dimensions of some quantities in mechanics with their units in bracket

- Velocity = displacement/time, its dimensions are L/T or LT^{-1} (ms^{-1})
- Acceleration : the dimensions are those of velocity/time that is L/T^2 or LT^{-2} (ms^{-2})
- Force, since force = mass * acceleration, its dimensions are MLT^{-2} ($kgms^{-2}$ or N)
- Work or Energy: since work = force * distance, its dimension is ML^2T^{-2} (Kgm^2s^{-2} or J)

Dimensions can be used to verify whether a physical equation is correct or not. The method of dimension can be used to find the relation between quantities when the mathematics is too difficult.

EXERCISE

1. Determine the dimension of the following physical quantities
   a. Density b. acceleration c. force d. work or energy e. power f. momentum
2. In the gas equation $(P+\frac{a}{v^2})(v-b) = RT$, what are the dimensions of the constants a and b

3. Show that the formula $S = Ut + ½(at^2)$ is dimensionally correct
4. A small mass is suspended from a long thread so as to form a simple pendulum. Suppose period T of the oscillations depends only on the mass m, the length l of the thread and the acceleration, g, of free-fall at the place concerned. Determine the exact relations between these quantities.
5. A wave is set up in a stretched string by plucking the string. Suppose the velocity C, of the wave depends on the tension F, in the string, its length l and its mass m, determine the relation between these quantities.

2.0 SCALARS AND VECTORS

2.1 Scalar

This is a physical quantity that has magnitude and is completely specified by a number and a unit. Examples are: mass, volume, frequency, length, time e.t.c. The rules of ordinary arithmetic are used to manipulate scalar quantities (symbols are represented in italic)

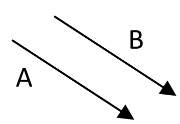
2.2 Vector

This is a physical quantity that has both magnitude and direction and is completely specific by a numerical number (value), a direction and a unit. Examples are: displacement, velocity, force e.t.c. A vector is represented analytically by a boldface type such as **F** when written by hand. When vector quantities are added, the direction must be taken into account.

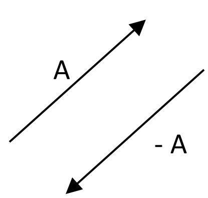
2.3 Vector Algebra

The following definitions are fundamental

1. Two vectors **A** and **B** are equal if they have the magnitude and directions regardless of their initial point that is **A**=**B**

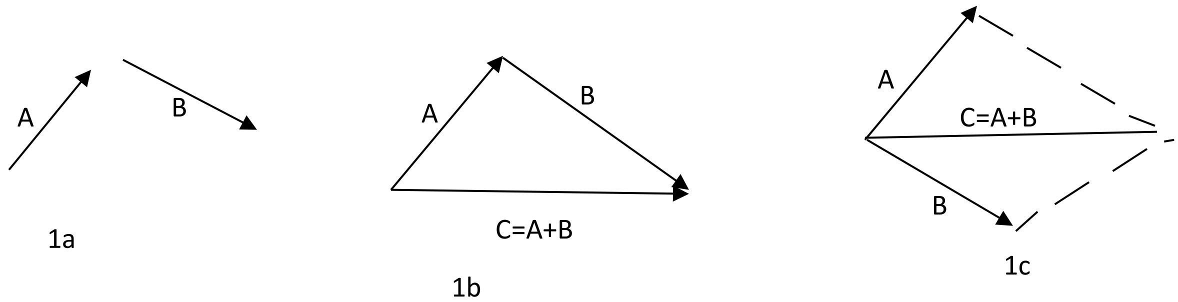


2. A vector having direction opposite to that of vector say **A** but with the same length is denoted as –**A.**



3. The sum of resultant of vectors **A** and **B** of figure 1(a) below is a vector C formed by placing the initial point of **B** on the terminal point of **A** and joining the initial point of **A** to

the terminal point of **B** (1b). We write **C**= **A**+**B**. This definition is equivalent to the parallelogram law for vector addition as indicated in fig 1c



For more than one vector: A, B, C, D = E (Resultant)

4. The difference of vectors **A** and **B** represented by **A-B** is that vector **C** which when added to **B** gives **A**. (that is **A** +(-**B**)). If **A**=**B** this implies that **A-B** = zero (null) vector. This has a magnitude of zero but its direction is not defined.
5. The product of a vector **A** by a scalar p is a vector p**A** or **A**p with magnitude p times the magnitude of **A** and direction the <u>same as</u> or <u>opposite</u> to that of **A** depending on whether p is positive (+ve) or negative (-ve). If p=0 this implies that p**A**=0 (null vector)

2.4 Laws of Vector Algebra

If **A** , **B** and **C** are vectors and p and q are scalars,

1. **A**+**B** = **B**+**A** — Commutative law for addition
2. **A** + (**B**+**C**) = (**A**+**B**) +**C** — Associative law for addition
3. p (q**A**) = pq**A** = q (p**A**) — Associative law for multiplication
4. (p+q)**A** = p**A**+q**A** — Distributive law
5. p(**A**+**B**) = p**A** + p**B** — Distributive law

2.5 Geometric Vector Addition

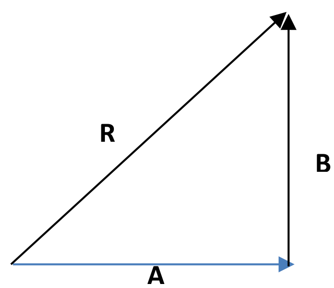
When two or more vectors are added together, those vectors must have the same unit. This can be done in number of ways

i. **Graphical method** – e.g fundamental 3- The order to which the vectors are added does not matter for three or more vectors. A geometric proof of this is (found) called associative law for additions. This method could also be called Geometric method. The rules for vector sums are conveniently described by geometric methods. To add vector **B** to vector **A**. First draw vector **A** on graph paper and then draw vector **B** with its tail starting from the tip (head) of **A**. Thus the resultant vector **R**= **A**+**B** is the vector drawn from the tail of **A** to the tip of **B**. This is Triangle method of Addition.

ii. An alternative graphical procedure for adding two vectors is known as the **PARALLELOGRAM rule of addition**. In construction, the tails of the two vectors **A** and **B** are together and the resultant vector **R** is the diagonal of parallelogram form with **A** and **B** as it sides. This leads to parallelogram law of vectors which states that: **if two vectors are represented in magnitude and direction by the adjacent side of a parallelogram, the resultant is represented in magnitude and direction by the diagonal of the parallelogram drawn from the common point (or tail) of the vectors** (figure 1c).

iii. Trigonometric Method

Although it is possible to determine the magnitude and direction of the resultant of two or more vectors of the same kind graphically with ruler and protractor, this procedure is not very exact. Thus for accurate results, its necessary to use trigonometry. This method is easy to find the resultant **R** of two vectors **A** and **B** that are perpendicular to each other. The magnitude of the resultant is given by the Pythagorean Theorem as **R** = $\sqrt{\boldsymbol{A}^2 + \boldsymbol{B}^2}$ and the angle ϴ between **R** and **A** may be found from tanϴ = **B**/**A**



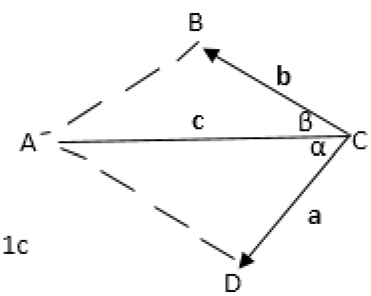


Fig 1c

From Figure 1c, apply cosine rule (formular), the magnitude of **C** which is the resultant or sum will be

$$C = (a^2 + b^2 \ - 2abcosB)^{1/2} \qquad (1)$$

When the vectors **a** and **b** act in the same direction (**a** and **b** are parallel) angle B will be 180^0 so that equation(1) gives the magnitude of **C** as

$$c = (a^2 + b^2 + 2ab)^{1/2} \qquad \text{Cos } 180^0 = -1$$

$$c = ((a + b)^2)^{1/2} = a + b$$

When **a** and **b** are in the opposite direction, angle B will be 0, then (**a** and **b** are antiparallel)

$$c = (a^2 + b^2 \ - 2abcos0)^{1/2} \qquad \text{Cos } 0^0 = 1$$

$$c = (a^2 + b^2 - 2ab)^{1/2}$$

$$c = ((a - b)^2)^{1/2} = a - b$$

If the two vectors are perpendicular, angle B = 90^0

$$c = (a^2 + b^2 \ - 2abcos\,90^0)^{1/2}$$

$$c = (a^2 + b^2\ )^{1/2} \qquad \textbf{a} \text{ is perpendicular to } \textbf{b}$$

And generally for **a** and **b** at an angle of 180-B to each other

$$c = (a^2 + b^2 \ - 2abcosB)^{1/2}$$

iv. Vector Addition: Component Method.

When vectors to be added are **not** perpendicular (3- dimensional cases) the method of addition by component is employed. The component of a vector in any given direction is the effect of that vector in the said direction.

Any vector **A** in 3- dimensions can be represented with initial point at the origin 0 of a rectangular coordinate system. Let A be the rectangular coordinates of the terminal vector **A** with initial point at O. The vectors A_1i A_2j A_3k are called the rectangular component vector (component vectors) of **A** in the x, y and z directions respectively. A_1, A_2 and A_3 are called the rectangular component (components) of A in the x, y and z directions respectively. The sum or resultant of $A_1 i, A_2 j\ and\ A_3 k$ is the vector **A** such that

$$A = \ A_1 i + A_2 j + A_3 k$$

The magnitude of A is: $A = I\boldsymbol{A}I = \sqrt{A_1^2 + A_{\mathbf{2}}^{\mathbf{2}} + A_3^2}$

In general the position vector or radius vector **r** from O to the point (x,y,z) is written as

$$r = \ xi + yj + zk$$

And has magnitude r= $\boldsymbol{IrI} = \sqrt{x^2 + y^2 + z^2}$

To add two or more vectors **A**, **B** and **C** by the component method follow this procedure

1. Resolve the initial vectors into components in the x, y and z directions
2. Add the components in the x direction to give Rx likewise y-direction to give Ry and the component in the z direction to give Rz. That is, the magnitude of Rx, Ry and Rz are given below respectively as

$$Rx = Ax + Bx + Cx \ldots\ldots\ldots\ldots\ldots\ldots$$

$$Ry = Ay + By + Cy \ldots\ldots\ldots\ldots\ldots\ldots$$

$$Rz = Az + Bz + Cz \ldots\ldots\ldots\ldots\ldots\ldots$$

Calculate the magnitude and direction of the resultant R from its component Rx, Ry and Rz by using the Pythagorean theorem :

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

If the vectors being added all lies in the same plane, only two components need to be considered.

v. Unit Vectors

A unit vector is a dimensionless vector of length unity used to specify a given direction. It has no other physical significance. They are used as a convenience in describing a direction in space. From figure 3, symbols i, j and k represent unit vectors pointing in the x,y and z direction

Exercises

1. A car travels 20.0km due north and then 35.0km in the direction 60 west of north . Find the magnitude and direction of the car's resultant displacement

2. A particle undergoes three consecutive displacement given by d_1 = (i + 3j-k)cm, d_2= (2i-j-3k)cm and d_3=(-i+j) find the resultant displacement of the particle

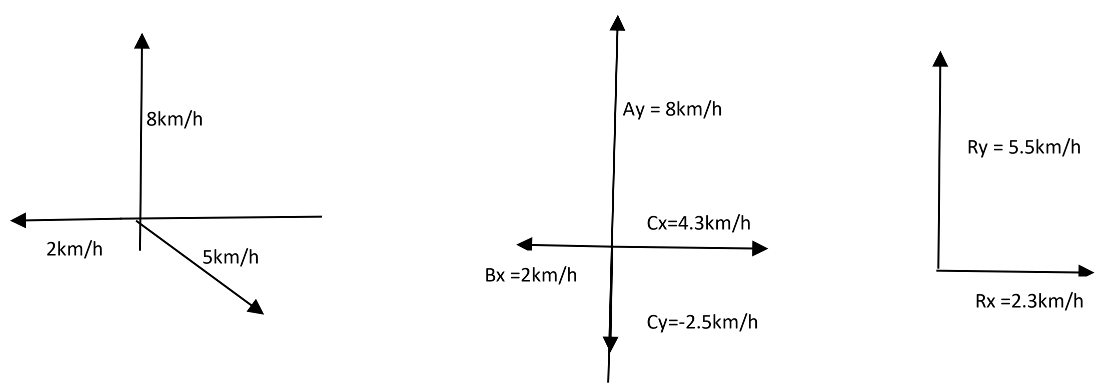
Solution.

R= $d_1+d_2+d_3$ = (2i+3j-4k)cm

The resultant displacement has components Rx = 2cm, Ry = 3cm and Rz = -4cm. its magnitude is

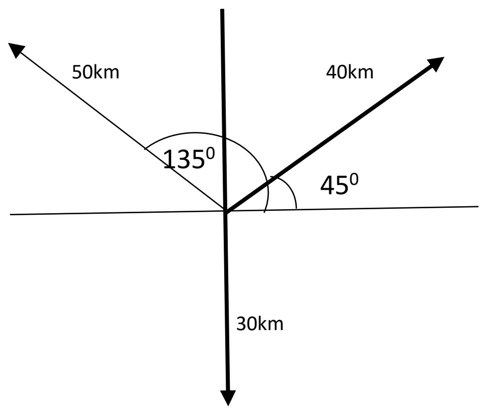
$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = 5.39cm$$

3. A boat headed north at a velocity of 8km/hr. A strong wind is blowing whose pressure causes it to move sideways to the west at a velocity of 2.0km/h . There is also a todal current that flow in a direction 30 south of east at a velocity of 5.0km/h . What is the boat's velocity relative to the earth surface



The direction Θ can be obtained by $tan\theta = \frac{Rx}{Ry} = 23^0$, $R = \sqrt{R_x^2 + R_y^2} = 6km$

4. A car is driven northeast for 40km, then northwest for 50km and then south for 30km. Draw the vector diagram and determine the resultant displacement of the car.



r = 34.38 km, 78.12^0N or

r(-7.08,33.64)

Tutorials

1. If **B** is added to **A,** under what condition does the resultant vector have a magnitude equal to A+B ? Under what condition is the resultant vector equal to zero?

2. Can the magnitude of particles displacement be greater than distance travelled?

3. If A=B, what can you conclude about their component of A and B?

4. Can the magnitude of a vector have a negative value? Explain

5. Can a vector have a component equal to zero and still have a non zero magnitude? Explain

6. Two vectors are given by **A**=3i-2j and **B**= -i -j
   Calculate (a) **A**+**B** , (b) **A**-**B** (c) |**A**+**B**|, (d) the direction of **A**+**B** and **A**-**B**

7. If one of the component of a vector is not zero its magnitude be zero? Explain

8. A particles undergoes the following consecutive displacements 3.5m south, 8.2m northeast and 15m west. What is the resultant displacement?

9. Find (a) graphically and (b) analytically the sum or resultant of the following displacements
   A = 2m NW, **B =** 4m 30^0 North of east , **C** = 7m due south

10. Two vectors are given by **A**= -2i + j -3k and **B** = 5i +3j-k (a) find a third vector **C** such that 3**A**+2**B**-**C**=0 (b) what are the magnitude of **A**, **B** and **C**

11. Two forces 10N and 6N acts on a body, the direction of the forces are not known (a) what is the maximum magnitude of these forces (b) what is the minimum magnitude? Ans. 16N, 4N

12. Vectors **A** and **B** have components Ax=-5cm, Ay=1.1cm, Az=-3.5cm and Bx = 8.8cm, By= -6.3cm, Bz= 9.2cm determine the components of the vectors (a) A+B , (b) B-A (c) 3B+2A (d) express the vector **B-A** in unit vector notation

13. Three forces acts at the point O as shown below find (a) the x and y components of the resultant force and (b) the magnitude and direction of the resultant force

3.0 VELOCITY AND ACCELERATION

Motion is characterized by a change in position. Motion in a straight path is called **linear** motion. Kinematics is a motion described using the concept of space and time regardless of the causes of the motions. The subject of dynamics on the other hand involves an object undergoing constant acceleration with the relationship between motion, forces and the properties of moving objects

3.1 Velocity

If a body moves from one point A to another point B in a time interval t, the length AB, represent the total change in position. This total change in position is called **displacement**, s, of a body in the direction of B. The unit of displacement is **metre**. On the average, the magnitude of the displacement changes by an amount s/t every one second. This amount is the magnitude of the average velocity of the body.

Velocity is defined as the time rate of change of displacement. If the displacement changes in equal amounts in equal time intervals, the velocity is said to be uniform or constant {that is v= s/t= constant}. If a particle moves along the x-axis from point P to point Q, Let its position at point P be x_i and time t and let its position at point Q be x_f at time t_f. At times other than t_i and t_f, the position of the particles between these two points may vary (that is explained better with position time graph). In the time interval $\Delta t = t_f - t_i$, the displacement of the particles is $\Delta x = x_f - x_i$

The average velocity of the particles V is defined as the ratio of the displacement Δx and the time interval Δt. It is independent of any path taken (e.g if a particles starts at a point and returns to the same point via any path, its average velocity is zero since Δx= 0). Displacement is not equal to distance covered (traveled). It can be positive or negative depending on the displacement.

The velocity of a particle at any instant of time or at some point on a space – time graph is called the **instantaneous velocity**. This concept is very important when the average velocity in different time intervals is not constant. The instantaneous velocity V equals the limiting value of the ratio Δx / Δt as Δt approaches zero.

$$V_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad \text{(Slope of the distance time graph)}$$

V can be positive, negative or zero. The instantaneous speed of a particle is defined as the magnitude of the instantaneous velocity. Note: speed can never be negative (-ve)

3.2 Acceleration

When the velocity of a particle changes with time, the particle is said to be accelerating. Acceleration is defined as the time rate of change of velocity. If the velocity of a body increases from an initial value, U (V_i) to a final value V (V_f) in a time interval, t, the average acceleration is given by

$$a = \frac{(V - U)}{t} = \frac{(v_f - v_i)}{(t_f - t_i)} = \frac{\Delta V}{\Delta t}$$

$$V = U + at \qquad (1)$$

The instantaneous acceleration is defined as the limit of the average acceleration as Δt approaches zero that is

$$a = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} = \frac{dV}{dt} \qquad \text{(Slope of the velocity time graph)}$$

Note: Retardation (-a) is rate of decrease of velocity

If a is positive (+ve), acceleration is in the +ve x-direction, -a simply implies that acceleration is in the negative (–ve) x-direction. This implies that Acceleration = instantaneous acceleration

If the velocity of the body increases by equal amounts in equal time interval, the acceleration is said to be uniform or constant that is

$$a = \frac{(V - U)}{t} = constant$$

The unit of V is m/s, a = m/s^2. If the velocity of a body increases uniformly from an initial velocity U to a final velocity V in a time interval t, the average velocity V_{avg} will be

$$V_{avg} = \frac{(V+U)}{2} = \frac{S}{t}$$

But from equation 1

$$\frac{S}{t} = \frac{(V+U)}{2} = \frac{((U+at)+U)}{2}$$

$$S = Ut + \tfrac{1}{2}at^2 \qquad (2)$$

If the body starts from rest, U=0. Equation 2 becomes

$$S = \frac{1}{2}at^2$$

Since from equation (1), V=U +at, then making t subject of the formular, we have that

$$t = \frac{(V-U)}{a}$$

Combining this equation with equation 2 eliminates t. the equation then becomes

$$S = U\left(\frac{V-U}{a}\right) + \frac{1}{2}a\left[\left(\frac{V-U}{a}\right)\right]^2$$

$$V^2 = U^2 + 2aS \qquad (3)$$

For U=0, equation 3 becomes

$$V^2 = 2aS$$ Or

$$V = \sqrt{2aS}$$

Equations (1), (2) and (3) are the three fundamental equations of motion for linear motion under constant acceleration or uniform acceleration.

3.3 Graphs of Motion

Graphs of motion for bodies under constant acceleration is of two types: Displacement-time graph (fig a) and Velocity-time graph. The slope of the displacement time graph of a body under constant acceleration is the velocity. That is

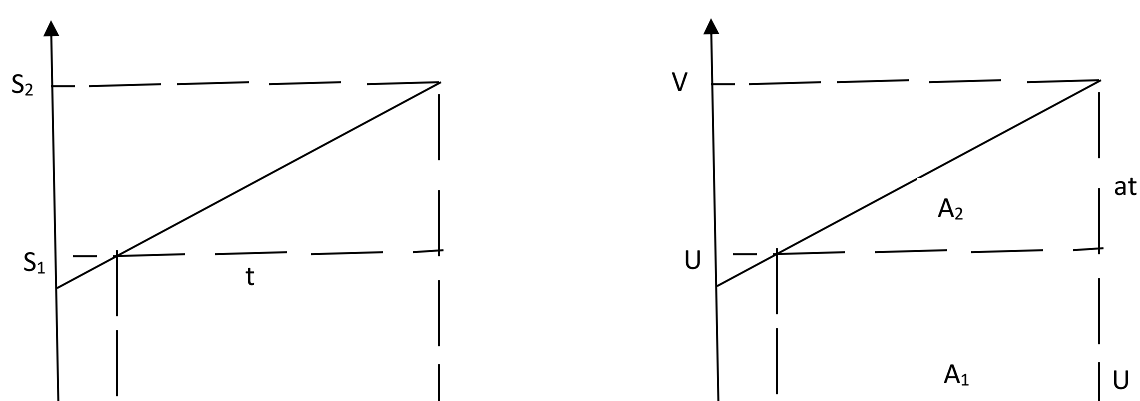
$$Slope = \frac{S_2 - S_1}{t_2 - t_1} = \frac{S}{t} = V \qquad where\ S = displacement$$

Figure (b) is the velocity – time graph of a body under constant acceleration. From the graph we see that:

$$Slope = \frac{(V - U)}{t} = a$$

The area A_1 of the rectangle is: $A_1 = Ut$ and the area A_2 of the triangle with base t is: $A_2=(1/2)at^2$. The total area, A under the velocity -time graph is

$$A = A_1 + A_2 = Ut + \frac{1}{2}at^2 = S(total\ displacement)$$



Examples

1. A car travels 270km in 4.5h (a) what is its average velocity (b) how far will it go in 7.0h at this average velocity (c) how long will it take to travel 300km at this average velocity.
   a. V_{avg}= ΔS/t = 270/4.5 = 6.0km/hr
   b. S =V_{avg}* t = 6.0*7 = 420km
   c. t = S/V = 300/60 =5.0h

2. A car travels at 100km/h for 2h, at 60km/h for the next 2h and finally at 80km/h for 1h. What is the car's average velocity for the entire journey.

   Average velocity=(total distance covered)/(total time taken)

   $$V_{avg} = \frac{(S_1+S_2+S_3)}{(t_1+t_2+t_3)} = \frac{(V_i t_1+V_2 t_2+V_3 t_3)}{(t_1+t_2+t_3)} = 80km/hr$$

   (S=Vt)

3. What is the acceleration of a car that goes from 20km to 30km/h in 1.5s (b) at the same acceleration, how long will it take the car to go from 30 to 36km/h
   a. Acceleration = (V-U)/t = 6.7 km/hr

   b. t= (V-U)/a = 0.9s (V= 36km, U=30km, a= 6.7km/hr)

4. The brake of a certain car can produce an acceleration of 6m/s2 (a) how long does it takes the car to come to from a velocity of 30m/s (b) how far does the car travel during the time the brake are applied.

a. t = V/a = 5sec

b. $S=Ut+(1/2)at^2$ = 75m U= 30m/s, t=5s

5. The brake of a car whose initial velocity is 30m/s are applied and the car receives an acceleration of -2m/s2. How will it have gone (a) when its velocity has decrease to 15m/s (b) when it has come to a stop.

a. $V^2 = U^2 + 2aS$

$$S = \frac{V^2 - U^2}{2a} = 169m$$

b. V=0. This implies that S=225m

6. The velocity of a particle moving along the x-axis varies with time according to the expression $v=(40-5t^2)$m/s when t is in Sec (a) find the average acceleration in the time intervals t=0 to t=2s (b) determine the acceleration at t=2sec

a. V_i coresponds to t_i hence $V_i = (40-5t_i^2)$; for $t_i=0$, $V_i=40$m/s

V_f corresponds to t_f hence $V_f = (40-5t_i^2)$; for t_f =2sec, V_f =20m/s

b. $average\ acceleration = \frac{\Delta V}{\Delta t} = \frac{V_f - V_i}{t_f - t_i} = -10m/s$

c. a=dV/dt at t=2. This implies -10t m/s hence is equal to $-20 m/s^2$

OR using limiting factor that is as $\Delta t \rightarrow 0$, the velocity at time t is given by $V = (40-5t^2)$ m/s and the velocity at time $t+\Delta t$ is given by

$$V = 40-5(t+\Delta t)^2 = 40 - 5t^2 - 10t\Delta t - 5(\Delta t)^2$$

The change in velocity over the time interval Δt is

$$\Delta V = V_f - V_i = [-10t\Delta t - 5(\Delta t)^2] \text{ m/s}$$

Divide through Δt and taking limit of the results as Δt approaches zero, we have acceleration at any time t as

$$a = \lim_{\Delta t \to 0}\left(\frac{\Delta V}{\Delta t}\right) = \lim_{\Delta t \to 0}(-10t - 5\Delta t) = -10t \qquad m/s$$

at t = 2sec, a = - 20m/s

TUTORIALS

1. An electron in a cathode ray tube of a TV set enters a region where it accelerates uniformly from a speed of 3×10^4 m/s to a speed of 5×10^6 m/s in a distance of 2cm
   (a) How long is the electron in the region where it accelerates
   (b) What is the acceleration of the electron in this region

1. Average veloc3and instantaneous velocity are generally different quantities, can they ever be equal for a specific type of motion? Explain
2. If V is non zero for some time interval Δt, does this mean that the instantaneous velocity is never zero during the interval? Explain
3. If the velocity of a particles is zero, can its acceleration ever be non zero and vice versa? Explain
4. A jogger runs in a straight line with an average velocity of 5m/s for 4min and then with an average velocity of 4m/s for 3min. (a) what is her total displacement (b) what is her average velocity during this time (1.92km, 4.57m/s)
5. At t=1s, a particle moving with constant velocity is located at x=-3m and at t=6s, the particle is located at x=5m,
   a. From this information, plot the position as a function of time
   b. Determine the velocity of the particles from the slope of this graph
6. The velocity of a particles moving along the x-axis varies in time according to the relation V= (15-8t) m/s. Find
   a. The acceleration of the particles
   b. It's velocity at t=3s
   c. It's average velocity in the time interval t=0 to t=2 s

      (-8m/s^2; -9m/s; 7m/s)

7. A particle travels in the +ve x-direction for 10s at a constant speed of 50m/s. It then accelerates uniformly to a speed of 80m/s in the next 5s. Find (a) the average acceleration of the particles in the first 10s (b) its average speed in the interval t=10s to t= 15s (c) the total displacement of the particles between t=0 and t=15s and (d) its average speed in interval t=10s to t=15s [0; 6m/s^2; 825m and 65m/s]

8. The initial speed of a body is 5.2m/s. What is its speed after 2.5s if it (a) accelerates uniformly at 3.0m/s^2 and (b) accelerates uniformly at -3.0m/s^2 (-x axis)? (12.7m/s; -2.3m/s)

9. An electron has an initial velocity of 3.0x10^5m/s. If it undergoes an acceleration of $8\text{x}10^{14}$m/s^2(a) how long will it take to reach a velocity of $5.4\text{x}10^5$m/s and (b) how far has it travelled in this time? ($3\text{x}10^{-10}$s; 1.26×10^{-4}m)

4.0 THE LAWS OF MOTION {Newton’s law of motion}

4.1 Force

The change in motion of an object is caused by force. If an object moves with uniform motion (constant velocity) its motion does not change, therefore no force is required to maintain the motion. If several forces act on an object (simultaneously), the object will accelerate only if the net force acting on it is not equal to zero (that is the resultant force or the unbalance force). If the net force is zero, the acceleration is zero and the velocity of the object remains constant. When the velocity of a body is constant or if the body is at rest, it is said to be in equilibrium. These forces can be **contact force** (pull and push) and **action-at-a-distance** for example (Gravitational, Electrostatic, magnetic)

Basically, there are four types of interactive force with both type above combined

I. Gravitational attractions between objects because of their mass
II. Electromagnetic forces between charges at rest or in motion
III. Strong nuclear forces between subatomic particles and
IV. Weak-nuclear forces (weak-interaction) arise as a result of radioactive decay processes

4.2 Newton's First Law and Inertial frame

The first law states that **"everybody continues in its state of rest or uniform motion in a straight line, unless impressed (external resultant force acts) force acts on it".** We can say that when the resultant force on a body is zero, its acceleration is zero. That is when F=0, then a=0 which implies that an isolated body is either at rest or moving with constant velocity.

This law is sometimes called **Law of Inertial**, since it applies to objects in an inertial frame of reference. An **inertial frame of reference** is one in which an object will move with constant velocity if left undisturbed. An inertial frame is one with no acceleration. In general, inertial frames are those in which Newton's laws of motion are valid.

The reluctance of a stationary object to move and the moving object to stop is called **Inertial.** Object with big mass feels more reluctant to change its state than an object with small mass which implies that mass of an object is a measure of its inertial.

4.3 Newton's Second Law and Fundamental Equation

The second law states that the change in momentum per unit time is proportional to the impressed force and it takes place in the direction of the straight line along which the force acts. In another way, the law states that the time rate of change of momentum of a particle is equal to the resultant external force acting in the object (particle).

$$\sum F = \frac{dP}{dt} = \frac{d(mV)}{dt} \quad (1)$$

$$p = mV \ (momentum\ of\ a\ particle)$$

$\sum F$ is the vector sum of all external forces. From equation (1), we have for m=constant,

$$\sum F = \frac{d(mV)}{dt} = m\frac{dV}{dt} \qquad but\ a = \frac{dV}{dt}$$

$$therefore, \quad \sum F = ma \qquad (2)$$

Equation (1) and (2) are Newton's 2nd law which is the **fundamental equation**

Therefore, from (2) we conclude that the resultant force on a particle equal to its mass multiplied by its acceleration if the mass is constant. Equation (1) is valid only when the speed of the particle is less than the speed of light.

The resultant force is zero if a=0, where V is constant or zero. Hence the 1st law of motion is a special case of the second law. The SI unit of force is N. A **force of 1N** is defined as the force that, when acting on a 1kg mass, produces an acceleration of 1m/s^2. 1N = 1 kgm/s. The table below shows different system of unit for mass, acceleration and force.

Systems of Units	**Mass**	**Acceleration**	**Force**
SI	Kg	m/s^2	N=kgm/s^2

Cgs	G	Cm/s^2	dyne = gcm/s^2
British Engineering	Slug	Ft/s^2	lb = slug ft/s^2

$1N = 10^5$ dyne = 0.225lb

Lb = pound: 1 pound of force when acting on a 1-slug mass produces an acceleration of $1ft/s^2$

1 Lb = 1 slug ft/s^2

4.4 Newton's third law

If two bodies interact, the force of body 1 and 2 (F_{12}) is equal to and opposite the force of body 2 on body 1 (F_{21}) (In a direction along the line joining the particles) then

$$F_{21} = -F_{12}$$

The action force is equal in magnitude to the reaction force and opposite in direction. that is the action and reaction forces always act in different objects. Note that an isolated force cannot exist in nature.

Examples

1. A force of 3000N is applied to a 1500kg car at rest (a) what is its acceleration (b) what will its velocity be 5s later.

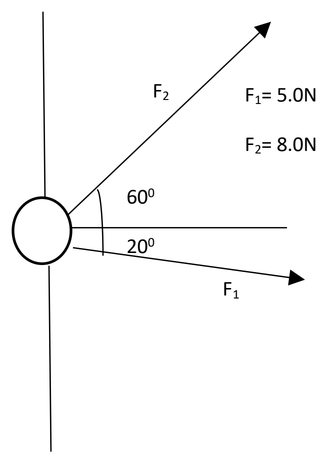
   (a) a = F/m = $2m/s^2$
   (b) v = at = 11.0m/s

2. A 60g tennis ball approaches a racket at 15m/s in contact with the racket for 0.005s and then rebounces at 20m/s. Find the average force that the racket exerted on the ball.

   The ball acceleration a = (V-U)/t
   V= -20m/s, U= 15m/s, t= 0.005s

F= ma = -420N (Minus sign shows that the force was in opposite direction)

3. An object of mass 0.3kg is placed on a horizontal frictionless surface. Two forces act on the object as shown in the figure below. The force F_1 has a magnitude of 5.0N and F_2 has magnitude of 8.0N. Determine the acceleration of the object



$$\sum F_x = F_{1x} + F_{2x} = F_1 cos20^0 + F_2 cos60^0 = 8.7N$$

$$\sum F_y = F_{1y} + F_{2y} = -F_1 sin20^0 + F_2 sin60^0 =$$

If a is +ve, $m_2 sin\Theta > m_1$ (that is m_2 accelerates down the inclined plane)

If $m_1 > m_2 sin\Theta$ (acceleration of m_2 is up the incline and downward for m_1)

$$a = \frac{(\Sigma F)}{m}$$

If we assume that m_1=10kg, m_2=5kg, $\Theta=45^0$, then a = -4.2m/s^2

The –ve sign shows that a is against gravity.

Tutorials

1. An object has mass of 200g. Find its weight in dyne and in N. Ans $1.96x10^5$ dyne or 1.96N

2. A person weighs 120lb. Determine (a) her weight in N (b) her mass in kg [534N & 54.4kg]
3. A 6kg object undergoes an acceleration of $2m/s^2$ (a) what is the magnitude of the resultant force acting on it ? (b) If this same force is applied to a 4kg object, what acceleration will it produce? [12N , $3m/s^2$]
4. A 4kg object has a velocity of 3i m/s at one instant. t seconds later, its velocity is (8i + 10j)m/s. Assuming the object was subject to a constant net force. Find a. the component of the force b. its magnitude (F_x=2.5N, F_y=5N)
5. Two forces F_1 and F_2 act on a 5kg mass. If F_1=20N, F_2=15N, find the acceleration in (a) F_1 perpendicular F_2

5.0 WORK, ENERGY & POWER

5.1 <u>Work</u>

WORK is defined as the product of displacement and force applied in the direction of the displacement. That is $\boldsymbol{W = FD}$. The work done by a constant force F acting on a particle is defined as the product of the component of the force in the direction of the particle's displacement and the magnitude of the displacement. that is the work done by F is : $\boldsymbol{W = Fscos\theta}$ [from F.S =

|FScosΘ|].However the work done by a varying force acting on an object moving along the x-axis from X_i to X_f is given by $\boldsymbol{W} = \int_{xi}^{xf} Fxdx$

If there are several forces acting on the particle, the net work done by the forces is the sum of the individual work done by each force.

5.2 Energy

Energy is defined as capacity for doing work, thus the unit of energy is same as work (that is Joule). Energy manifest in different forms, these are:

1. Kinetic energy such as the energy possess by a moving car or a falling stone due to its motion
2. Chemical energy such as the energy contain in food from which derives energy used while working
3. Heat energy such as the energy developed in the engine of a car which the car uses for motion
4. Electrical energy: energy travelling along conductors used in driving fans
5. Light energy: energy emitted by light bulbs
6. Sound energy: energy in speakers
7. Nuclear energy: energy derive from the nucleus of atoms, nuclear generators

Energy in one form can be transform into another form by means of appropriate mechanisms.

5.2.1 Mechanical energy

A body can possess mechanical energy or have the ability to do mechanical work either by virtue of its motion or its position. For example, a car moving with velocity **V** possess K.E by virtue of its motion while a stone held at a height, h possess P.E by virtue of its position.

5.2.2 Kinetic Energy

Supposed a particle has a constant mass and that at times t_1 and t_2, it is located at point p_1 and p_2 and moving with velocities $v_1 = \frac{dv_1}{dt}$ and $v_2 = \frac{dv_2}{dt}$ respectively. Therefore, the total work done in moving the particle from p_1 to p_2 is given by $w = \int F.dv = \frac{1}{2}m({v_1}^2 - {v_1}^2)$

Total work done from p_1 to p_2 → K.E at p_2 , - K.E at p_1

$$\rightarrow \qquad w = T_2 - T_1 \text{------------------ (1)}$$

Where $T_1 = {}^1\!/_2\, m{v_1}^2$; $T_2 = {}^1\!/_2\, m{v_2}^2$ $\qquad (V << C)$

Kinetic energy is defined as the energy a body possesses by virtue of its motion.

5.2.3 <u>Potential Energy</u>

The scalar V such that $F = -\nabla v$ is called the potential energy or scalar potential of the particle in the conservative force field.

The total work done from $p_1 \ to\, p_2 \ \rightarrow P.E\ at\ p_1 - P.E\ at\ p_2$ can be written as

$$w = v_1 - v_2 \text{ --------------------------- (2)} \qquad v_1 = v(p_1)$$

Energy needed in some machine is stored in some components of the machine as **potential energy**. In a pure mechanical system only these two forms of mechanical energy are considered with the result that in energy conservation law, the total energy gives

Total Energy= K.E + P.E = constant (for a pure mechanical system)

$E = T + V = constant$ (Principle of conservation)

The **work-energy theorem** states that **"the net work done on a particle by external forces equals the change in K.E of the particle"**

That is $\qquad W = k_f - k_i = {}^1\!/_2\, m{v_f}^2 - {}^1\!/_2\, m{v_i}^2$

5.3 Power

The time rate of doing work on a particle is often called the **instantaneous power** or **power applied to the particle.**

$$P = \frac{dw}{dt} \qquad \text{where W = work; P = power}$$

If **F** is the force acting on a particle and **V** is the velocity of the particle, then

$$P = F.V = \frac{work\ done\ (or\ expended)}{time} = \frac{w}{t}$$

The unit of power is Joule per second J/s or watt (w). 1watt =1 J/s (1hp = 746watts =0.75kw)

5.4 Efficiency ŋ

Efficiency **ŋ** is defined as the ratio of the useful energy (or work) output of a machine (or any other energy converting system) to the energy (or work) input. That is

$$ŋ = \frac{\textbf{energy output}}{\textbf{energy input}}$$

The efficiency of any real machine is less than unity (one) because the energy output e_o is always less than the energy input e_i. The missing energy dissipates in the machine due to dissipative factors (energy) so that

$$E_i = E_o + E_d \quad or \quad W_i = W_o + W_d$$

$$thus \qquad ŋ = \frac{E_o}{E_i} = \frac{W_o}{W_i} = \frac{P_o}{P_i} \quad (P_o \quad P_i \ \rightarrow power\ input\ /\ output)$$

5.5 Work done by a Spring

The Spring will exert a force on the body (attached) given by $F_s = -Kx$ [x = displacement; K = force constant. The negative sign (-ve) indicates that F_x is in opposite direction to x]. Since the spring force always acts toward the equilibrium position, it is sometimes called **Restoring force.**

Work done from $x_1 = x_0 \; to \; x_f = 0$, implies that

$$\int_{x_1}^{x_f} F_s \; dx = \int_{-x_m}^{0} (-Kx)dx$$

Work done WD by the spring is positive (+ve)

$$\text{WD} = {}^{1}\!/_{2} \, kx_m^2 \qquad (1)$$

WD by the applied force (external agent) is given by

$$WD_{Fapp} = \int_o^{x_m} F_{app} dx = \int_o^{x_m} Kxdx = {}^{1}\!/_{2} \, kx_m^2$$

<u>Examples</u>

1. A ball of mass 0.1kg is thrown vertically upward with an initial velocity of 20m/s. Calculate the (a) P.E halfway (b) P.E at max height (c) P.E as it leaves the ground (d) K.E half way up (e) K.E as it leaves the ground (f) K.E at the max height and use your answer to show that energy is conserved in this exercise.

   a. $V^2 = U^2 - 2gh$ at h_{max}, V=0

      $U^2 = 2gh$

      h_{max} = 20.4m

      Half way up (${}^{1}\!/_{2} \, h$ =10.2m)

      P.E = 10J

   b. $P.E_{max}$ = 20J

   c. P.E at ground level

      h=0 then P.E =0J

   d. K.E at the ground level V= 20m/s

      K.E=${}^{1}\!/_{2} mV^2$ = 20J

e. $K.E_{1/2h} = {}^{1}/_{2}mV^{2}$=10J

f. K.E at h_{max} that is at V=0, K.E = 0J

For energy to be conserved, the total mechanical energy (sum of K.E and P.E) must be the same for all points or height. Thus for the 3 heights

P.E + K.E = (0+20)J = (10+10)J= (20+0)J= 20J

2. A concrete slabs weighing 1500N each are to be loaded into a trailer which is 1.5m high. The loader used an inclined plane made of plank Incline at an angle of 15degree to the horizontal. If the coefficient of dynamic friction between the plank and the slap is 0.3, calculate the efficiency of the inclined plane.

Work input W_i = Work done by the loafers = FS

F = Component of weight along the incline + dynamic frictional force

F = W sin 150 + $R_x u$

But R_x=Wcos 150, u = 0.3, W=1500

F = 822.9N

S = length of the inclined plane = 1.5/(Sin 15^0) = 5.80 m

W_i = FS = (822.9 x 5.8) J

= 4772.79N

Work output = WD by inclined plane = W_o

= mgh = wh = (1500 x 1.5) J

$$the\ efficiency = \frac{work\ output}{work\ input} = \frac{w_o}{w_i} = \frac{1500\ x\ 1.5}{822.9\ x\ 5.8} = 0.472 = 47.2\%$$

3. A 6kg block initially at rest is pulled to the right along horizontal, smooth surface by a const. Horizontal force of 12N. Find the speed of the block after it moves a distance of 3m

Solution:

The WD by this force is:

$$W_f = F_s = (12N)(3m) = 36N.m = 36J$$

Using the work- energy theorem that is $W_f = K_f - K_i = \frac{1}{2}mv_{f^2} - 0 \rightarrow V_f = 3.46m/s$

Or from $V^2 = U^2 + 2as$

$$\rightarrow \left(a = \frac{2m}{s} from\ F = ma\right) \rightarrow V = 3.46m/s$$

4. A block of mass 1.6kg is attached to a spring with a force const. Of 10^3 N/m. The spring is compressed a distance 2.0cm and the block is released from rest (a) calculate the velocity of the block as it passes through equilibrium position X=0 if the surface is frictionless (b) calculate the velocity of the block through equilibrium position for constant frictional force of 4N retards it motion.

Solution:

$X_m = -$ 2cm= -2×10^{-2}m

$W_s = \frac{1}{2}kx^2{}_m = 0.2J \ \ (k = 10^3 N/m)$

Using work energy theorem with $V_i = 0$ gives

$$W_s = \frac{1}{2}mv_f{}^2 - \frac{1}{2}mv_i{}^2 v_i = 0$$

$$v_f = 0.5\ m/s$$

(b) $w_f = -f_s = -(4N)(2\ x10^{-2}m) = -0.08J\ WD\ by\ the\ frictional\ force$

The net WD on the block = WD by the spring + WD by friction

$.i.e\ W_{net} = W_s + W_f = 0.12J$ Using work energy theorem gives:

$$\frac{1}{2}mv^2{}_f = W_{net}$$

$$V_f = 0.39\ m/s$$

TUTORIAL

1. Can the K.E of an object have a –ve value explain
2. If the speed of a pole is double, what happens to its K.E
3. What can be said about the speed of an object if the net work (W_{net}) on that object is zero
4. A 15kg block is dragged over a rough horizontal surface by a constant force of 70N acting at an angle of 25^otot the horizontal. the lock is displaced 5m by (a) the 70N force (b) the force of friction (c) the normal force (d) the force of gravity (e) the net work done on the body [Ans: a. 317J b. -176J c. zero d. zero e. 141J]
5. A force acting on a particle is given by F_x = (8x - 16), where x is in meters. Find the net work done by the force as the particle moves from x = 0 to x = 3m.
6. A 3kg mass has an initial velocity of V_0 = (5i-3j)m/s a. what is its kinetic energy at this time? B. find the change in its Kinetic energy if its velocity changes to (8i+4j)m/s [hint V^2 = **V.V**] [Ans: 51J, b. 69J]
7. A mechanic pushes a 200kg car from rest to a speed of 3m/s with a constant horizontal force. During this time, the car moves a distance of 30m. neglecting friction between the car and the road, determine a. the work done by the mechanic b. the horizontal force exerted on the car [ans: 9000J b. 300N]
8. A 1500kg Car accelerates uniformly from rest to a speed of 10m/s in 3secs. Find the a. work done on the car in this time b. the average power delivered by the engine in the first 3sec c. the instantaneous power delivered by the engine at t=2sec [Ans: 75000J b. 25000W (33.5hp) c. 33300W (44.7hp)]
9. Is any work being done on a car moving with constant velocity along a straight level road? Explain.
10. A car of mass 800kg travelling at 70km/h on a horizontal road is brought to rest by the action of the brakes in a distance of 15m. calculate the amount of work done in stopping the car. If the co-efficient of dynamic friction between the tyres and the road is 0.2, calculate the average retarding force due to the brakes

11. In 1970, the population of the world was about $3.5*10^{20}$ and about $2*10^{20}$J of work was performed under human condition. Find the average power consumption per person in watts and horse power? [1yr = $3.15*10^{7}$S] [1.8 KW; 2.4 hp]
12. An 800kg car moving at 6m/s begins to move downhill 40m high with its engine off. The driver applies the brakes so that the car's speed at the bottom of the hill is 20m/s. how much energy s lost to friction?
13. A book weighs about 1.5lb. What is the rest energy in foot-pounds? In Joules?
14. A boy pulls a wagon with a force of 45N by means of a rope that makes an angle of 40^{0} with the ground. How much work does he do in moving the wagon 50m? (Ans; 1.72 KJ)
15. A 15kg object initially at rest is raised to a height of 8m by a force of 200N. What is the velocity of the object at this height? [Ans: 7.5m/s]